Quantum Reflection of Ultracold Atoms in Magnetic Traps

Rui-Hua Xie and Paul Brumer[a]

Max-Planck-Institute für Strömungsforschung, Bunsenstr.10, D-37073 Göttingen
[a] Chemical Physics Theory Group, University of Toronto, Toronto M5S 3H6, Canada

Reprint requests to Dr. R.-H. X., E-mail: rxie@gwdg.de

Ultracold neutral atoms can be trapped in spatially inhomogeneous magnetic fields. In this paper, we present a theoretical model and demonstrate by using Landau-Zener tool that if the magnetic resonant transition region is very narrow, "potential barriers" appear and quantum reflection of such ultracold atoms can be observed in this region.

The Landau-Zener model [1] has become a standard notion in quantum physics, and has been extensively studied during the recent years [2]. It provides the probability of transition between two quantum states coupled by an external field of a constant amplitude and a time-dependent frequency which passes through resonance with the transition frequency. The level crossing, which is seen in the diabatic basis (i.e., the basis of the two bare states-the eigenstates of the Hamiltonian in the absence of interaction), appears as an avoided crossing in the adiabatic basis (i.e., the basis comprising the two eigenstates of the Hamiltonian in the presence of interaction). There are a number of cases of level crossings and avoided crossings, which can be met in quantum physics, solid state physics, molecular physics, magnetic resonance, atomic collisions, atom-surface scattering, and nuclear physics. Really, the Landau-Zener model is a reliable qualitative and even quantative tool for describing and understanding such phenomena.

Recent experimental developments enable precise manipulation of cold atoms by lasers [3, 4]. Small and accurate velocities of the atoms can be achieved using advanced cooling [5, 6] and launching [7] techniques, and a detuned laser field can be used to create controlled and adjustable potentials for the atoms [4, 8]. Under these conditions, the quantum nature of the dynamics may become important [9]. Indeed, quantum tunneling of atoms has recently been observed [10].

It is known that ultracold neutral atoms can be trapped in spatially inhomogeneous magnetic field. In this paper, we present a theoretical model and demonstrate, by using the Landau-Zener tool, that if the magnetic resonant transition region is very narrow, "potential barriers" appear and quantum reflection of such ultracold atoms (i.e., above-barrier, classically forbidden reflection of atoms) can be observed in this region.

We assume that an ultracold atom with spin $1/2$ propagates along the z-axis in the positive direction. It is subject to a gradient magnetic field, $\boldsymbol{B}_g = B_g z\hat{z}$, and at the same time an oscillating magnetic field couples the two spin states. Here we assume that the oscillating field is circularly polarized, $\boldsymbol{B}_o = B_o(\cos\omega t\hat{x} + \sin\omega t\hat{y})$. The gradient field acts similarly to the field in a Stern-Gerlach experiment, in that as a result of the gradient field, the difference in potential between the spin states changes linearly along the z axis. At the point $z_0 = \omega/\gamma B_g$ at which the spin states differ in potential by $\hbar\omega$ (with some broadening due to uncertainty), the coupling field induces magnetic resonant transitions between the two spin states. The transition region is proportional to the amplitude ratio between the oscillating and gradient fields, i.e., B_o/B_g. Ignoring any electric polarization effects, and taking into account magnetic dipole interaction, the Hamiltonian for the atom is given by:

$$H(z,t) = \frac{p_z^2}{2m} - M(\boldsymbol{B}_g + \boldsymbol{B}_o) \tag{1}$$

$$= -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} - \gamma S[B_g z\hat{z} + B_o\cos\omega t\hat{x} + B_o\sin\omega t\hat{y}]$$

$$= -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} - \frac{\hbar\gamma}{2}\begin{pmatrix} B_g z & B_o e^{-i\omega t} \\ B_o e^{i\omega t} & -B_g z \end{pmatrix},$$
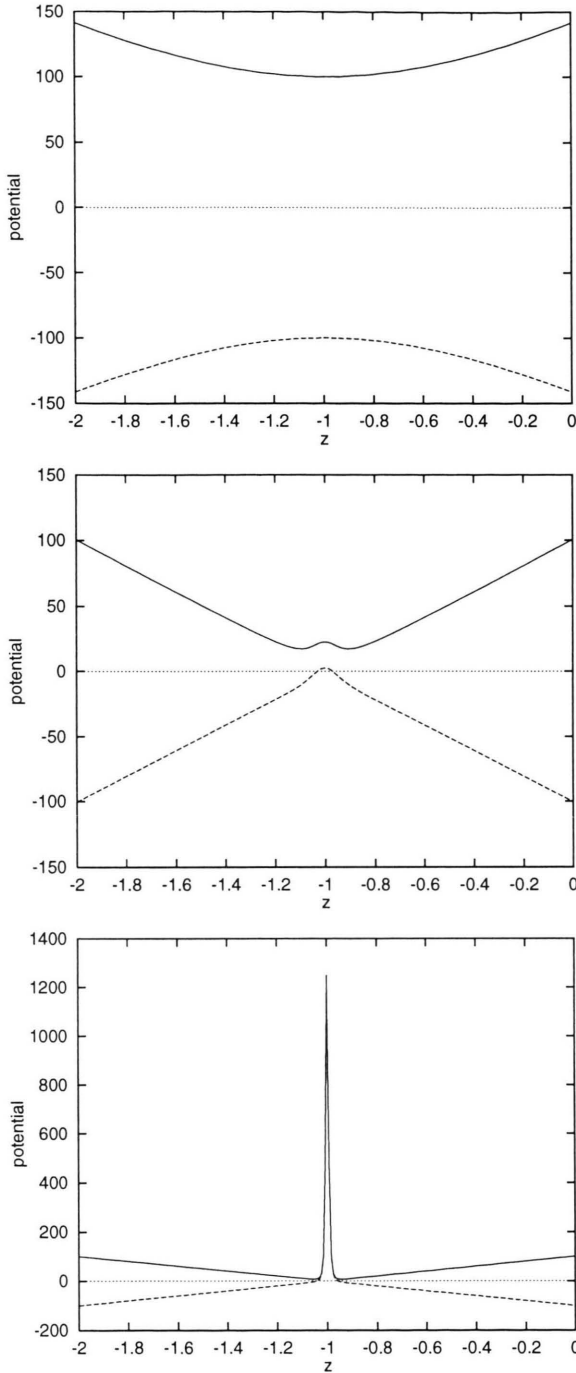
Fig. 1. The potential V_+ (solid line) and V_- (dashed line) with $\gamma = -2.0$, $\hbar = 1$, $m = 1$, $\omega = 200$, $B_g = 100.0$, and (a) $B_o/B_g = 1$; (b) $B_o/B_g = 0.1$; (c) $B_o/B_g = 0.01$.

where S is the spin of the atom and γ is the gyromagnetic ratio. Defining Ψ_+ and Ψ_- as the spacial wave-

functions of spin up and down states, respectively, we get a Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \Psi_+(z,t) \\ \Psi_-(z,t) \end{pmatrix} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} \begin{pmatrix} \Psi_+(z,t) \\ \Psi_-(z,t) \end{pmatrix} \tag{2}$$

$$-\frac{\hbar\gamma}{2} \begin{pmatrix} B_g z & B_o e^{-i\omega t} \\ B_o e^{i\omega t} & -B_g z \end{pmatrix} \begin{pmatrix} \Psi_+(z,t) \\ \Psi_-(z,t) \end{pmatrix}.$$

Then, with the transformation

$$\Phi_+(z,t) = \Psi_+(z,t)\, e^{i\omega t/2}, \tag{3}$$
$$\Phi_-(z,t) = \Psi_-(z,t)\, e^{-\omega t/2} \tag{4}$$

we have

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \Phi_+(z,t) \\ \Phi_-(z,t) \end{pmatrix} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} \begin{pmatrix} \Phi_+(z,t) \\ \Phi_-(z,t) \end{pmatrix} \tag{5}$$

$$+ \begin{pmatrix} U_+ & D \\ D & U_- \end{pmatrix} \begin{pmatrix} \Phi_+(z,t) \\ \Phi_-(z,t) \end{pmatrix},$$

where the coupling term $D = -\hbar\gamma B_o/2$, and the potential U_+ and U_- are given by

$$U_+ = -\frac{\hbar}{2}\gamma B_g z + \frac{\hbar}{2}\omega, \tag{6}$$

$$U_- = \frac{\hbar}{2}\gamma B_g z - \frac{\hbar}{2}\omega. \tag{7}$$

It is clear that $U_+ = U_- = 0$ at the resonant point z_0, i.e., both potential curves in the diabatic base cross at the resonant point.

Finally, taking the diabatic-adiabatic transformation [1, 2]

$$\begin{pmatrix} \phi_+(z,t) \\ \phi_-(z,t) \end{pmatrix} = T \begin{pmatrix} \Phi_+(z,t) \\ \Phi_-(z,t) \end{pmatrix}, \tag{8}$$

where

$$T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \tag{9}$$

and

$$\sin 2\theta = \gamma B_o / \sqrt{(-\gamma B_g z + \omega)^2 + (\gamma B_o)^2} \le 0, \tag{10}$$

we get the Schrödinger equation in the adiabatic base

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \phi_+(z,t) \\ \phi_-(z,t) \end{pmatrix} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} \begin{pmatrix} \phi_+(z,t) \\ \phi_-(z,t) \end{pmatrix} \quad (11)$$

$$+ \begin{pmatrix} V_+ & E \\ -E & V_- \end{pmatrix} \begin{pmatrix} \phi_+(z,t) \\ \phi_-(z,t) \end{pmatrix},$$

where the coupling term $E = -\frac{\hbar^2}{2m}[2(\partial\theta/\partial z)(\partial/\partial z) + (\partial^2\theta/\partial z^2)]$, and the potentials V_+ and V_- are given by

$$V_+ = \frac{\hbar^2\gamma^4 B_0^2 B_g^2}{8m[(-\gamma B_g z + \omega)^2 + (\gamma B_0)^2]^2}$$
$$+ \frac{\hbar}{2}\sqrt{(-\gamma B_g z + \omega)^2 + (\gamma B_0)^2}, \quad (12)$$

$$V_- = \frac{\hbar^2\gamma^4 B_0^2 B_g^2}{8m[(-\gamma B_g z + \omega)^2 + (\gamma B_0)^2]^2}$$
$$- \frac{\hbar}{2}\sqrt{(-\gamma B_g z + \omega)^2 + (\gamma B_0)^2}, \quad (13)$$

Based on the above potentials in the adiabatic base, we find that $V_+ = \frac{\hbar^2 B_g^2}{8m B_0^2} + \frac{\hbar|\gamma|B_0}{2}$, and $V_- = \frac{\hbar^2 B_g^2}{8m B_0^2} - \frac{\hbar|\gamma|B_0}{2}$ at the resonant point z_0. Both potential curves in the adiabatic base do not cross at the resonant point z_0. Figures 1 (a) - (c) show the potential V_+ and V_- for different ratios between B_0 and B_g. It is found that, if the transition region is very narrow, for example, for the case of $B_0/B_g = 0.01$, sharp potential barriers are observed in both potential curves in the transition region. It is known that there exists quantum reflection of atoms due to such potential barriers. Exactly, the quantum reflection probability curves for those potentials can be got by numerical simulation.

The quantum effect, over-barrier reflection, is in fact dominated by the potential regions where the semiclassical treatment fails. As an example, in case
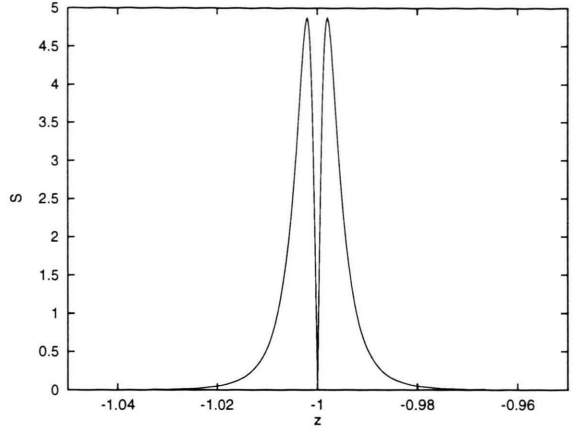

Fig. 2. The badlands associated with the potential V_- in Fig. 1(c), where $E = 1.2V_-(z = z_0)$.

of the potential V_- of Fig. 1 (c), the de Broglie wavelength varies slowly when the distance is far away from the resonant transition point z_0, where

$$S = \frac{1}{2\pi}\left|\frac{d\lambda}{dz}\right| = \frac{\hbar}{\sqrt{8m(E - V_-)^3}}\left|\frac{dV_-}{dz}\right| \ll 1, \quad (14)$$

$$\frac{dV_-}{dz} = \frac{\hbar^2\gamma^5 B_0^2 B_g^3}{2m}\frac{(-\gamma B_g z + \omega)}{[(-\gamma B_g z + \omega)^2(\gamma B_0)^2]^3}$$
$$+ \frac{\hbar\gamma B_g}{2}\frac{(-\gamma B_g z + \omega)}{\sqrt{(-\gamma B_g z + \omega)^2 + (\gamma B_0)^2}}, \quad (15)$$

but there are two "badlands" beside the resonant point z_0, where the Wentzel-Kramers-Brillouin (WKB) approximation breaks down. In Fig. 2, such badlands are shown for the potential V_- of Fig. 1 (c) with $E = 1.2V_-(z = z_0)$.

In conclusion, using Landau-Zener tool, we have shown that, if the magnetic resonant transition region is very narrow, a "potential barrier" appears and quantum reflection of atoms can be observed in this region.

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